

Universal Chaotic Scattering on Quantum Graphs

Z. Pluhař^a and H. A. Weidenmüller^{b*}

^a*Faculty of Mathematics and Physics, Charles University, 180 00 Praha 8, Czech Republic*

^b*Max-Planck-Institut für Kernphysik, 69029 Heidelberg, Germany*

We calculate the S -matrix correlation function for chaotic scattering on quantum graphs and show that it agrees with that of random-matrix theory (RMT). We also calculate all higher S -matrix correlation functions in the Ericson regime. These, too, agree with RMT results as far as the latter are known. We conclude that our results give a universal description of chaotic scattering.

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Purpose. Closed quantum systems that are chaotic in the classical limit possess universal spectral fluctuation properties. Depending on symmetry, these coincide with the fluctuation properties of one of Dyson's three canonical random-matrix ensembles [1]. These statements, originally formulated in the form of a conjecture [2], have since been demonstrated [3, 4] with the help of Gutzwiller's semiclassical periodic-orbit theory for the two-point level correlator of general chaotic systems and, in the case of orthogonal symmetry, for chaotic quantum graphs [5].

For open chaotic quantum systems, the fluctuation properties of the scattering matrix (S -matrix) are at issue, quantified in terms of the totality of S -matrix correlation functions. As for closed systems, it would be desirable to establish (at least) the complete equivalence between the S -matrix two-point correlation function for chaotic scattering with that of random-matrix theory (RMT) (calculated in Ref. [6] for the case of orthogonal symmetry and in Ref. [7] for the case of unitary symmetry). However, we are not aware of any analytical results for S -matrix correlations for chaotic scattering.

In the present Letter, we fill that gap. For the case of orthogonal symmetry, we calculate the S -matrix two-point correlation function analytically for open chaotic quantum graphs. We show that our result coincides with that [6] of random-matrix theory (RMT). We also calculate all higher S -matrix correlation functions in the Ericson regime. These coincide with RMT results as far as the latter are known [6, 8, 9]. In that regime, the S -matrix elements are supposed [10, 11] to have a Gaussian distribution. Our results show that this is the case only for strong absorption in all channels. We argue that our results are universal, i.e., apply to chaotic scattering in general. This establishes the complete equivalence of chaotic scattering and the random-matrix approach to scattering of Ref. [6].

We focus attention on chaotic quantum graphs because here the semiclassical expansion is exact and scattering theory is particularly transparent. Chaotic scattering on quantum graphs was introduced in Refs. [12, 13] where

many of its properties were displayed with the help of numerical simulations, see also Refs. [14].

Scattering Matrix. We largely follow the developments of Ref. [13]. A graph is a system of V vertices labelled α, β, \dots that are linked by B bonds. For simplicity of notation we assume that every vertex α is linked by a single bond $(\alpha\beta)$ to every other vertex β ("completely connected graph"). Then the number of bonds is $B = V(V-1)/2$. Our results remain valid, however, if some bonds are missing and B is smaller than $V(V-1)/2$ as long as the graph remains connected. We consider the limit $V \gg 1$. The lengths L_b of all bonds $b = (\alpha\beta)$ are assumed to be incommensurate but similar. That is the first necessary assumption for the graph to be chaotic. One vertex or some or all V vertices are linked by a single bond each (a "lead") to infinity. The number Λ of such leads obeys $1 \leq \Lambda \leq V$ and defines both, the number of scattering channels and the dimension of the S -matrix.

On each bond or lead, waves propagate freely with wave number k (the same for all bonds/leads), and the wave function is a linear combination of amplitudes $\exp\{ikx\}$ and $\exp\{-ikx\}$ where x is the distance to one of the vertices attached to the bond/lead. For all bonds/leads, the coefficients of the linear combination are determined by boundary conditions specified in terms of V matrices $\Gamma^{(\alpha)}$ of dimension V defined for each vertex α . The matrix $\Gamma^{(\alpha)}$ expresses the outgoing amplitudes on the lead and on each of the $V-1$ bonds attached to vertex α (written symbolically as \mathcal{O}) in terms of the incoming amplitudes on the same or any other bond or the lead attached to α (written symbolically as \mathcal{I}) so that $\mathcal{O} = \Gamma^{(\alpha)}\mathcal{I}$. With $\beta, \gamma \neq \alpha$ the unitary and symmetric matrix $\Gamma^{(\alpha)}$ has the form

$$\Gamma^{(\alpha)} = \begin{pmatrix} \rho^{(\alpha)} & \tau_{\gamma}^{(\alpha)} \\ \tau_{\beta}^{(\alpha)} & \sigma_{\beta\gamma}^{(\alpha)} \end{pmatrix}. \quad (1)$$

Here $\rho^{(\alpha)}$ describes backscattering on lead α , $\tau_{\beta}^{(\alpha)}$ describes scattering from bond $(\alpha\beta)$ to lead α or vice versa, and $\sigma_{\beta\gamma}^{(\alpha)}$ describes scattering from bond $(\alpha\beta)$ to bond $(\alpha\gamma)$ or vice versa. In general, the matrix $\sigma^{(\alpha)}$ is symmetric and subunitary. For a vertex α without lead the first row and column of $\Gamma^{(\alpha)}$ are lacking, we have $\Gamma^{(\alpha)} = \sigma^{(\alpha)}$, and $\sigma^{(\alpha)}$ is symmetric and unitary.

Given a single incident wave in channel α only, these

*Electronic address: Hans.Weidenmueller@mpi-hd.mpg.de

boundary conditions completely define the total wave function. The amplitude of the outgoing wave in channel β is the element $S_{\alpha\beta}(k)$ of the symmetric and unitary scattering matrix. To write $S_{\alpha\beta}(k)$ explicitly, we define the symmetric subunitary block-diagonal matrix Σ of dimension $2B = V(V-1)$. Each of the V diagonal blocks of dimension $(V-1)$ carries one of the matrices $\sigma^{(\alpha)}$, $\alpha = 1, \dots, V$. All other elements of Σ vanish. This defines the “vertex representation” $\Sigma^{(V)}$ of Σ . The “bond representation” $\Sigma^{(B)}$ is obtained by a reordering of rows and columns. We assign to every bond $(\alpha\beta)$ the direction $d = +$ ($d = -$) if $\alpha > \beta$ ($\alpha < \beta$, respectively). We arrange the B bonds with positive (negative) direction in lexicographical order and label them consecutively with a running index $b = 1, \dots, B$. Then every directed bond is uniquely defined by (b, d) . The total number of directed bonds is $2B$. The map $\sigma_{\beta\gamma}^{(\alpha)} \rightarrow \sigma_{(\alpha\beta), (\alpha\gamma)}$ defines the bond representation of $\sigma^{(\alpha)}$ and, thus, the bond representation $\Sigma^{(B)}$ of Σ . The map $\tau_{\beta}^{(\alpha)} \rightarrow \tau_{(\alpha\beta)}$ similarly defines the bond representation of the vector \mathcal{T} . In bond representation we define the diagonal matrix $\exp\{-ik\mathcal{L}\}$ with elements $\delta_{bb'}\delta_{dd'}\exp\{-ikL_b\}$. Diagonal elements in locations that differ only in the sign of d are pairwise equal. Written somewhat symbolically the S -matrix is

$$S_{\alpha\beta}(k) = \delta_{\alpha\beta}\rho^{(\alpha)} + (\mathcal{T}\mathcal{W}^{-1}\mathcal{T})_{\alpha\beta} \quad (2)$$

where $\mathcal{W} = \exp\{-ik\mathcal{L}\} - \Sigma^{(B)}$. Expanding \mathcal{W}^{-1} in powers of $\Sigma^{(B)}$ we obtain a simple physical interpretation of Eq. (2). The term containing the n th power of $\Sigma^{(B)}$ is the sum of all semiclassical trajectories that connect the vertices α and β via passage through $(n+1)$ bonds. Each of the traversed bonds (bd) yields the factor $\exp\{ikL_b\}$.

Averages. The average over k is indicated by angular brackets and is taken over an interval of k -values that is large compared to π/L_{\min} where L_{\min} is the smallest bond length. Because of the incommensurability of the L_b , that average is equivalent [5] to a phase average: For any function F we have $\langle F[\exp\{ikL_b\}] \rangle = (1/(2\pi)) \int_0^{2\pi} d\phi_b F[\exp\{i\phi_b\}]$. Then Eq. (2) implies $\langle S_{\alpha\beta} \rangle = \delta_{\alpha\beta}\rho^{(\alpha)}$ [13] and can, thus, be read as $S = \langle S \rangle + S^{\text{fl}}$ where the fluctuating part is $S^{\text{fl}} = \mathcal{T}\mathcal{W}^{-1}\mathcal{T}$. It also follows that the average of the product of any number of S -matrix elements is equal to the product of the averages (as in RMT). The RMT analogue of $\langle S_{\alpha\alpha} \rangle = \rho^{(\alpha)}$ reads [6] $\langle S_{\alpha\alpha} \rangle = (1-x_\alpha)/(1+x_\alpha)$. Here x_α depends on the density of resonances; such dependence is conspicuously absent for graphs where $\rho^{(\alpha)}$ is independent of V .

Supersymmetry and saddle-point approximation. The S -matrix correlation function (P, Q) is defined as the average of a product of P elements of S^{fl} with arguments $k + \kappa_p$, $p = 1, \dots, P$ and Q elements of $S^{\text{fl}*}$ with arguments $k - \tilde{\kappa}_q$, $q = 1, \dots, Q$. Without loss of generality we assume $P \geq Q \geq 1$. Since $S^{\text{fl}} = \mathcal{T}\mathcal{W}^{-1}\mathcal{T}$, it suffices to work out

$$\left\langle \prod_{p=1}^P \mathcal{W}_{b_p d_p, b'_p d'_p}^{-1}(k + \kappa_p) \prod_{q=1}^Q (\mathcal{W}_{b_q d_q, b'_q d'_q}^{-1}(k - \tilde{\kappa}_q))^* \right\rangle. \quad (3)$$

We generalize the approach of Ref. [5] (where only the (1,1) correlator was considered). Using supersymmetry [6, 15], the correlator (3) is written as the $(P+Q)$ -fold derivative of the average of a generating function G (a superintegral). The average over k is calculated as a phase average over all ϕ_b with the help of the colour-flavour transformation [16] in its most general form (for $P \neq Q$). Integrating out the original integration variables gives

$$\langle G \rangle = \int d(\tilde{Z}, Z) \exp\{-\mathcal{A}(\tilde{Z}, Z)\} \quad (4)$$

where $\mathcal{A}(\tilde{Z}, Z)$ is the action

$$\begin{aligned} \mathcal{A}(\tilde{Z}, Z) = & -\text{STr} \ln(1 - Z\tilde{Z}) + \frac{1}{2} \text{STr} \ln(1 - ZzZ^\tau z) \\ & + \frac{1}{2} \text{STr} \ln(1 - \mathcal{B}_+^{-1} \tilde{Z}^\tau z (\mathcal{B}_-^\dagger)^{-1} \tilde{Z} z). \end{aligned} \quad (5)$$

Here STr is the supertrace. All matrices are defined in bond and in retarded-advanced representation. The matrix Z (\tilde{Z}) fills the upper (lower) non-diagonal block of the retarded-advanced representation, respectively, and is explicitly given by $Z_{bdpts, b'd'qt's'} = \delta_{bb'} Z_{b; dpts, d'qt's'}$, and correspondingly for \tilde{Z} . Here (b, d) are defined above, p (q) label the blocks needed to accommodate the P (Q) matrices \mathcal{W} (\mathcal{W}^*) in the correlator (3), respectively, $t = 1, 2$ is the time-reversal index as in Ref. [5], and $s = \mathbf{f}, \mathbf{b}$ refers to commuting (anticommuting) variables, respectively. The matrices Z and Z^τ are related as in Ref. [5]. For $P > Q$, Z and \tilde{Z} are rectangular matrices. With $a = \pm$ distinguishing the retarded and the advanced sector, the matrix z is $\delta_{aa'}\delta_{bb'}\delta_{dd'}\delta_{tt'}\delta_{ss'}(\delta_{a+\delta_{pp'}}\exp\{i\kappa_p L_b/2\} + \delta_{a-\delta_{qq'}}\exp\{i\tilde{\kappa}_q L_b/2\})$. The matrix \mathcal{B}_+ (\mathcal{B}_-) is only defined for the retarded (the advanced) sector, respectively, and is given by $(\mathcal{B}_+^{-1})_{bdpts, b'd'p't's'} = \delta_{tt'}\delta_{ss'}\delta_{pp'}(\Sigma_{bd, b'd'}^{(B)} + \sigma_3^s A^{(p)})$, and correspondingly for \mathcal{B}_-^{-1} . Here σ_3^s is the third Pauli matrix in superspace, and $A^{(p)}$ denotes the source term needed to generate by differentiation of G the matrix element $\mathcal{W}_{b_p d_p, b'_p d'_p}^{-1}$. The integration measure in Eq. (4) is the flat Berezinian. Up to this point our results are exact.

We calculate $\langle G \rangle$ using the saddle-point approximation, putting $z = 1$ and $A^{(j)} = 0$ for all $j = p, q$. As in Ref. [5], variation of the resulting action with respect to Z and \tilde{Z} yields $Z^\tau = \tilde{Z}$ and, with $\Sigma^{(B)} = (\Sigma^{(B)})^\tau$, the saddle-point equation $(1 - Z\tilde{Z})^{-1}Z = (1 - \Sigma_+ Z \Sigma_-^* \tilde{Z})^{-1} \Sigma_+ Z \Sigma_-^*$. Here Σ_+ is given by $\delta_{a+\delta_{tt'}}\delta_{ss'}\delta_{pp'}\Sigma_{bd, b'd'}^{(B)}$, and correspondingly for Σ_- . The saddle-point equation holds if $Z\Sigma_-^* = \Sigma_+^* Z$ and if $\Sigma^{(B)}\Sigma^{(B)*} = 1$. We use the vertex representation $\Sigma^{(V)}$ defined above. Eq. (7) below shows that all eigenvalues but one of every matrix $\sigma^{(\alpha)}$ filling one of the diagonal blocks of $\Sigma^{(V)}$ have magnitude unity. Except for corrections of order $1/V$ worked out below we, therefore, have $\Sigma^{(V)}\Sigma^{(V)*} = 1$ and, thus, $\Sigma^{(B)}\Sigma^{(B)*} = 1$. To satisfy $Z\Sigma_-^* = \Sigma_+^* Z$ we follow Ref. [5] and write the saddle-point solution Y as $\delta_{bb'}\delta_{dd'}Y_{pts, qt's'}$, and correspondingly

for \tilde{Y} . The matrix $\Sigma^{(V)}$ is closest to the generic case if all eigenvalues of $\Sigma^{(V)}$ differ from each other. That is the second necessary assumption for the quantum graph to be chaotic.

Corrections to the saddle-point action are due to deviations from $z = 1$, and from $\Sigma^{(B)}\Sigma^{(B)*} = 1$. Concerning the former, we assume $|\kappa_p|L_b \ll 1$, $|\tilde{\kappa}_q|L_b \ll 1$ for all p, q, L_b , expand [5] z and the action \mathcal{A} up to first order in κ_p and $\tilde{\kappa}_q$, use $\Sigma^{(V)}\Sigma^{(V)*} = 1$, and drop the source terms. With $\langle d_R \rangle = (1/\pi) \sum_b L_b$ the average level density [5], the first-order term in the exponent of Eq. (4) (the “symmetry-breaking term”) is

$$SY = i\pi \langle d_R \rangle \left(\text{STr}_{pst} \frac{\kappa}{1 - Y\tilde{Y}} + \text{STr}_{qst} \frac{\tilde{\kappa}}{1 - \tilde{Y}Y} \right). \quad (6)$$

The trace is only over the subspaces indicated, the matrix κ is $\delta_{ss'}\delta_{tt'}\delta_{pp'}\kappa_p$, and correspondingly for $\tilde{\kappa}$. Deviations from $\Sigma^{(B)}\Sigma^{(B)*} = 1$ are calculated by putting $z = 1$ and dropping the source terms. We use Eq. (1), suppress the index α , and take ρ to be real (as in RMT, that suppresses all elastic scattering phase shifts). Since Γ is unitary and symmetric it can be unitarily transformed into

$$\begin{pmatrix} \rho & \exp\{-i\phi_1\}T^{1/2} & 0 \\ \exp\{-i\phi_1\}T^{1/2} & -\rho \exp\{-2i\phi_1\} & 0 \\ 0 & 0 & \delta_{\mu\nu} \exp\{i\phi_\mu\} \end{pmatrix}. \quad (7)$$

Here $T = 1 - \rho^2$ is the transmission coefficient, the indices μ, ν run from 2 to $V - 1$, and the phases ϕ_1, ϕ_μ are real and arbitrary. Eq. (7) shows that $\sigma\sigma^*$ differs from the unit matrix only in the first diagonal element which is $1 - T$. Using that for all channels α we obtain for the correction term in the exponent of Eq. (4) (the “channel-coupling term”)

$$CH = -\frac{1}{2} \sum_{\alpha=1}^V \text{STr}_{pst} \ln \left(1 + T^{(\alpha)} \frac{Y\tilde{Y}}{1 - Y\tilde{Y}} \right). \quad (8)$$

Vertices not coupled to a lead do not contribute to the sum. In Ref. [5] it was shown that for $V \gg 1$ the massive modes do not contribute. That statement applies also in the present, more general case. Collecting everything we find

$$\langle G \rangle = \int d(Y, \tilde{Y}) \left(\dots \right) \exp\{SY + CH\}. \quad (9)$$

The term in big round brackets contains the source terms.

Two-point Function. For the correlation function (1, 1) the matrices Y and \tilde{Y} are both square matrices of dimension four, and it is straightforward to work out the source terms in Eq. (9). Lack of space does not permit us to present any details. Suffice it to say that using the transformations $t_{12} = Y(1 - Y\tilde{Y})^{-1/2}$, $t_{21} = \tilde{Y}(1 - Y\tilde{Y})^{-1/2}$, writing $\langle S_{\alpha\alpha} \rangle$ for $\rho^{(\alpha)}$ and $T^{(\alpha)} = 1 - |\langle S_{\alpha\alpha} \rangle|^2$, and replacing the wave-number arguments of S by energies, the resulting expressions for $\langle S_{\alpha\beta}^{\text{fl}}(k + \kappa) S_{\gamma\delta}^{\text{fl}*}(k - \tilde{\kappa}) \rangle$ become formally identical to the corresponding terms in Eq. (7.23)

of Ref. [6] for all values of the number $\Lambda = 1, \dots, V$ of channels. For the terms SY and CH in Eqs. (6) and (8) that can be checked directly. In the source terms, the phase ϕ_1 cancels out. These facts establish the equivalence of the two-point functions of RMT and of chaotic scattering on quantum graphs.

Ericson Regime. That regime is defined by the condition $\sum_{\alpha} T^{(\alpha)} \gg 1$. The cross section for chaotic scattering is expected to display Ericson fluctuations [10, 11]. Numerical simulations [13] have confirmed that expectation. Eq. (9) allows us to determine the leading terms in an asymptotic expansion in inverse powers of $\sum_{\alpha} T^{(\alpha)}$ of all (P, Q) -correlation functions and, thus, the complete distribution of S -matrix elements in the Ericson regime. The asymptotic terms are obtained [17] by keeping in Eq. (9) only terms of lowest order in Y, \tilde{Y} . For $SY + CH$ we obtain

$$-\frac{1}{2} \sum_{pq} \left(\sum_{\alpha} T^{(\alpha)} - 2i\pi \langle d_R \rangle (\kappa_p + \tilde{\kappa}_q) \right) \text{STr}_{st} (Y_{pq} \tilde{Y}_{qp}). \quad (10)$$

For the two-point function, the calculation [17] yields

$$\langle S_{\alpha\beta}^{\text{fl}}(k + \kappa) S_{\gamma\delta}^{\text{fl}*}(k - \tilde{\kappa}) \rangle = \frac{(\delta_{\alpha\gamma} \delta_{\beta\delta} + \delta_{\alpha\delta} \delta_{\beta\gamma}) T^{(\alpha)} T^{(\gamma)}}{\sum_{\tau} T^{(\tau)} - 2i\pi \langle d_R \rangle (\kappa + \tilde{\kappa})}. \quad (11)$$

With $T^{(\alpha)} = 1 - |\langle S_{\alpha\alpha} \rangle|^2$ and the replacement of wave numbers by energies, this is exactly the expression obtained for RMT in Refs. [8, 18]. For the general (P, Q) correlation function we need to find the leading-order contribution to the source terms. We expand the last term in Eq. (5) with $\tilde{Z}^{\tau} \rightarrow Y$ and $\tilde{Z} \rightarrow \tilde{Y}$, retaining only terms linear in Y and \tilde{Y} . (Only these are of the form $\sum_{pq} Y_{pq} \tilde{Y}_{qp}$ which, according to Eq. (10), gives the leading-order contribution). We need P source terms from \mathcal{B}_+ . Expanding the exponential we keep the term $(1/(2^P P!)) [\sum_{pq} \text{STr}_{st} ((\mathcal{B}_+^{-1})_p Y_{pq} (\mathcal{B}_+^{\dagger-1})_q \tilde{Y}_{qp})]^P$. No two source terms in \mathcal{B}_+^{-1} may have the same labels. That gives $(1/2^P) \prod_p [\sum_q \text{STr}_{st} ((\sigma_3^s A^{(p)} Y_{pq} (\mathcal{B}_+^{\dagger-1})_q \tilde{Y}_{qp})]$. Since all source terms in $\mathcal{B}_+^{\dagger-1}$ must also be different, the sum over q goes for $P = Q$ over all permutations of $q = 1, \dots, Q$. Equivalently we may keep all q fixed and sum over all permutations of $p = 1, \dots, P$. For $P > Q$ there are $P - Q$ source terms $A^{(p)}$ in $\mathcal{B}_+^{\dagger-1}$ that do not have a counterpart in $\mathcal{B}_-^{\dagger-1}$. For these $\mathcal{B}_-^{\dagger-1}$ is replaced by Σ . Each of the resulting supertraces corresponds to one of the factors in the flat integration measure $d(Y, \tilde{Y}) = \prod_{pq} d(Y_{pq}, \tilde{Y}_{qp})$. Therefore and because of Eq. (10), each superintegral factorizes into PQ terms, each factor characterized by the pair (p, q) of indices. In factors that do not carry any source terms the superintegration gives unity. The integration over those supertraces which carry both factors $A^{(p)}$ and $A^{(q)}$ yields the asymptotic form (11) of the average of a pair of S -matrix elements. For the $P - Q$ unpaired source terms $A^{(p)}$ the superintegration gives a non-vanishing contribution for $\alpha_p = \beta_p$ only. The result-

ing factor is

$$\mathcal{F}_{\alpha_p}(\kappa_p) = - \sum_{q=1}^Q \frac{T^{(\alpha_p)} \langle S_{\alpha_p \alpha_p} \rangle}{\sum_{\gamma} T^{(\gamma)} - 2i\pi(\kappa_p + \tilde{\kappa}_q) \langle d_R \rangle} \quad (12)$$

where again the phase ϕ_1 in Eq. (7) cancels out. The sum over q arises because in the advanced block, the matrix $\mathcal{B}_-^{\dagger-1}$ carries the same entry Σ in every block labelled q . We suppress all arguments k for brevity. That gives

$$\left\langle \prod_{p=1}^P S_{\alpha_p \beta_p}^{\text{fl}}(\kappa_p) \prod_{q=1}^Q S_{\alpha'_q \beta'_q}^{\text{fl}*}(-\tilde{\kappa}_q) \right\rangle = \sum_{\text{sel}} \prod_{j=1}^{P-Q} \mathcal{F}_{\alpha_{p_j}}(\kappa_{p_j}) \\ \times \sum_{\text{perm}} \prod_{q=1}^Q \left\langle S_{\alpha_j \beta_j}^{\text{fl}}(\bar{\kappa}_j) S_{\alpha'_q \beta'_q}^{\text{fl}*}(-\tilde{\kappa}_q) \right\rangle. \quad (13)$$

The sum with index “sel” goes over all $\binom{P}{P-Q}$ possibilities to select $(P-Q)$ matrix elements S^{fl} from the first factor on the left-hand side. These give rise to the first product which vanishes unless all selected elements are diagonal. The remaining Q elements S^{fl} , symbolically written as $S_{\alpha_j \beta_j}^{\text{fl}}(\bar{\kappa}_j)$ with $j = 1, \dots, Q$, appear as first factors in the angular brackets on the right-hand side. The sum with index “perm” extends over all permutations of these elements. Each of the terms in angular brackets on the right-hand side is equal to the asymptotic form (11) of the two-point function.

Eq. (13) gives the asymptotic form of all S -matrix correlation functions (P, Q) and, thus, the complete distribution of the k -dependent scattering matrix in the Ericson regime. For the $(2, 2)$ correlation function it agrees with the result of Ref. [8]. If $\langle S_{\alpha\alpha} \rangle = 0$ for all α (strong absorption in all channels), the factors \mathcal{F} all vanish, the correlation functions (P, Q) vanish for $P \neq Q$, and for $P = Q$ have the form characteristic of a Gaussian random process. In particular, all elements of S have a Gaussian distribution centered at zero, and cross-section

fluctuations have the form predicted in Refs. [10, 11]. If $\langle S_{\alpha\alpha} \rangle \neq 0$ in some channel α , that fact and the unitarity constraint $|S_{\alpha\alpha}| \leq 1$ distort the Gaussian distribution of $S_{\alpha\alpha}$. This is the cause of the occurrence of the factor \mathcal{F}_{α} (Eq. (12)). The factor shows that the distortion is biggest for $|\langle S_{\alpha\alpha} \rangle| = 1/\sqrt{2}$. For cross-section correlation functions, interest is focussed on the $(2, 1)$ correlation function. It was first noted in Refs. [9] that this function differs from zero (the result given there agrees asymptotically with our Eq. (13)). Implications of that fact for cross-section fluctuations have been discussed in Ref. [19].

Conclusions. For a quantum graph to be chaotic, the lengths L_b of all bonds must be incommensurate, and all eigenvalues of Σ must differ. For chaotic scattering on a quantum graph, we have derived formal analytical expressions for all (P, Q) correlation functions of the S -matrix. These were used to show that the $(1, 1)$ correlation function is identical to the one obtained from RMT, and to calculate for all (P, Q) explicit expressions in the Ericson regime. The latter agree with RMT results as far as these are known and yield the complete S -matrix distribution function in that regime. It may perhaps be possible to obtain from Eq. (9) explicit expressions also for $(2, 1)$ and $(2, 2)$. Our results apply to every chaotic quantum graph since we have used wave-number averages (not ensemble averages).

The agreement of our results and those of RMT on the one hand and of the two-point function for closed graphs [5] and that for general closed chaotic systems [4] on the other, strongly suggest that our results are universal, i.e., apply to chaotic quantum scattering in general. That implies that chaotic scattering and the RMT approach to scattering are completely equivalent.

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